

Research with Undergraduates

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The purpose of doing research with undergraduates.

- * Enhance students' interest in mathematics and mathematical research.
- * Show students the creative aspects of mathematics study.
- * Show students what mathematicians do.
- * Help students to decide whether they want to further study mathematics.

How to select and recruit students?

- * Summer REU program supported by NSF, students from other universities, 8 weeks.
- * William and Mary summer research program, William and Mary students, 6 to 8 weeks.
- * Research during the academic year supported by NSF, William and Mary students, one to two semesters.
- * Honors projects of William and Mary mathematics majors, two semesters.

How to select research topics?

- * The problem can be described easily, and there are special cases that students can try.
- * The problem can be approached by different techniques; there are endless possibilities that students can further their study.
- * The problem has connections to other areas.
- * The students may have problems in their minds.

If not, matrix theory is a good area!!!

- * Indeed, I have done research projects with students on coding theory, game theory, mathematical biology based on their requests.

You want to do something you like.

You also want to do something the students like!!!

Examples

Preserver problems

Let $M, N \in M_n$ with $\det(MN) = 1$. Define $\phi : M_n \rightarrow M_n$ by

$$\phi(A) = MAN \quad \text{for all } A \in M_n, \text{ or}$$

$$\phi(A) = MA^tN \quad \text{for all } A \in M_n.$$

Then ϕ is linear and

$$\det(A) = \det(\phi(A)) \quad \text{for all } A \in M_n.$$

Questions

- * How about the converse? That is, are there any other linear preservers of determinant?

Answer is affirmative by a result of [Frobenius, 1897].

- * How about weaken the hypothesis, say, just assume ϕ is additive,
 ϕ satisfies $\det(\phi(A) + \phi(B)) = \det(A + B)$ for all A, B ,
 ϕ satisfies $\det(\phi(A) - \phi(B)) = \det(A - B)$ for all A, B , etc.

More questions

- * How about other matrix invariant?
- * How about preservers of controllable systems and observable systems?
- * How about preservers of (Perron root) growth rate of a mathematical population models?

Asking the right questions is important in doing research and generating new knowledge.

How do we work?

- * Decide a topic, and I will provide some necessary background.
- * Meet everyday to discuss old and new ideas.
- * Do calculation on M_2, M_3, \dots
- * Try computer experiments.
- * Try some approaches I know.
- * Try some approaches I do not know!
- * Students may insist on understanding some concepts.
- * Students may make some interesting observations.
- * Students may carry out detailed computation that shows the pattern of the behaviour.
- * One should make good use of the talents of students.

Results of the projects

- * We solved problems, and published papers.
- * We learned more (interesting) mathematics.
- * Most of my students moved on to graduate study with better preparation - how to pose problems, approach problems, and solve problems.
- * I am glad to show more people the beauty of mathematics.
- * More importantly, I have let more people know that mathematicians are not crazy and boring, and they are doing interesting and useful things! Do you agree?

Visit my webpage – <http://www.math.wm.edu/~ckli>
and send me your questions – ckli@math.wm.edu

THANK YOU FOR YOUR ATTENTION!