Expanding the Lotka-Volterra Predator-Prey Model

Edward Baumann

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LYNX

- The Lynx is the predator
- 2’ - 3.5’ in length
- Lives less then 20 years in the wild

Hare

- The Hate is the prey
- 16” - 20” in length
- Lives less then 2 years in the wild

Assumptions

A few assumptions have to be made:

- The two populations only interact with each other
- Three subpopulations for each group (Juvenile, Mature, Post-Mature)
- Individuals born as Juveniles have to progress through each each group in the correct order
- Juveniles will be sheltered by adults
- Mature adults are the most fit

Starting Equations

Lotka-Volterra Predator-Prey Model

$$\frac{dH}{dt} = a * H - b * H * L$$

$$\frac{dL}{dt} = -c * L + d * H * L$$

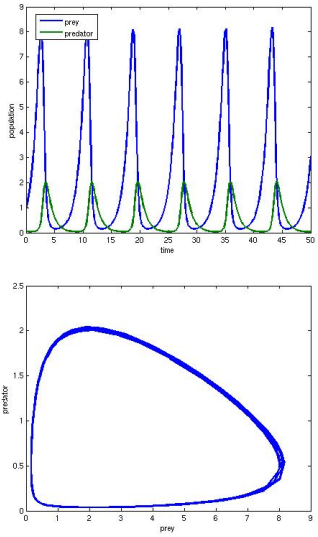

Figure 1: Graphed Solution

Leslie Matrix Model

$$L = \begin{bmatrix} s_1 m_1 & s_1 m_2 & \dots & s_1 m_{w-1} & s_1 m_w \\ s_2 & 0 & \dots & 0 & 0 \\ 0 & s_3 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & s_w & 0 \end{bmatrix}$$

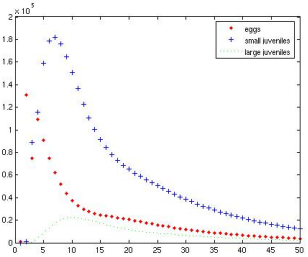

Figure 2: Loggerhead Solution

Combining the Models

In the Lotka-Volterra model, let H= $\begin{matrix} H_j \\ H_m \\ H_p \end{matrix}$ and L= $\begin{matrix} L_j \\ L_m \\ L_p \end{matrix}$

Plugging these values into the Lotka-Volterra model we find the following equations:

$$\frac{dH_j}{dt} = m2 * H_m * s1 + m3 * H_p * s1 - b * H_j$$

$$\frac{dH_m}{dt} = b*s2*H_j - d*H_m - M*H_m*L_m - N*H_m*L_p$$

$$\frac{dH_p}{dt} = d*s2*H_m - k*H_p - O*H_p*L_m - P*H_p*L_p$$

$$\frac{dL_j}{dt} = m5 * s4 * L_m + m6 * s4 * L_p - \frac{g * L_j}{H_j + H_m + H_p}$$

$$\frac{dL_m}{dt} = g * s5 * L_j - \frac{i * L_m}{H_j + H_m + H_p}$$

$$\frac{dL_p}{dt} = i * s6 * L_m - \frac{l * L_p}{H_j + H_m + H_p}$$

Graphical Solution

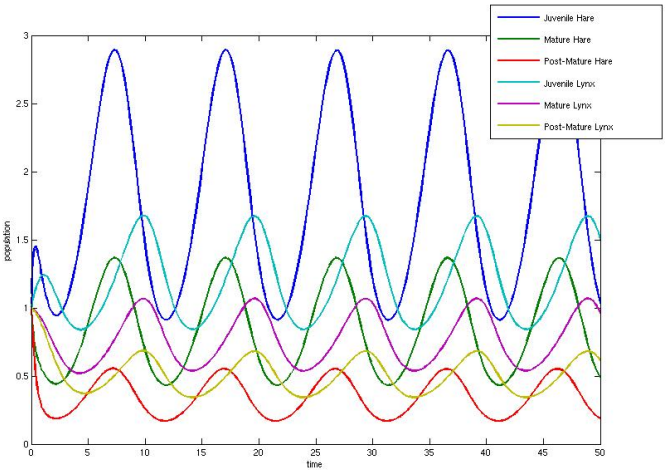

Figure 3: Cyclic Populations

Graphical Solution

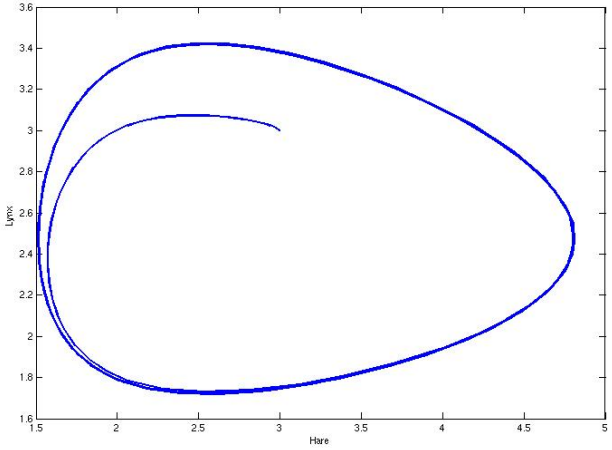

Figure 4: Cyclic Populations

Graphical Solution

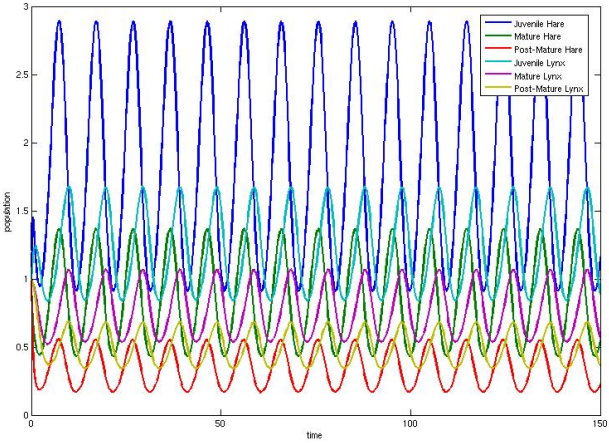

Figure 5: A Longer Time Scale