Assignment 1

Math 345, Prof. Shi

Due: Tuesday, Sept 12 (9:30am)

1. The following data taken from the 1996 Information Please Almanac show U.S. population from 1800 to 1860, just prior to the Civil War.

year	1800	1810	1820	1830	1840	1850	1860
population in million	5.31	7.24	9.64	12.87	17.07	23.19	31.44

   Use linear regression to find the best linear model describing the data. (First use Matlab program, then use the formula given in class, and compare the results.)

2. Populations may be harvested for a variety of reasons including economic gain (e.g. harvesting fish to sell), conservation (e.g. regulating wildebeest populations to prevent overgrazing), and public health (e.g. reducing mosquito abundance to reduce the incidence rate of the West Nile virus). There are many forms of harvesting models and we consider one of the simplest here. Consider a population that reproduces annually and whose population size in the beginning of the n-th year is x_n. Let r denote the number of progeny produced per year per individual and h denote the number of individuals harvested per year. If harvesting occurs occurs after reproduction and before the beginning of a new year, then $x_{n+1} = rx_n - h$. Solve the linear difference equation $x_{n+1} = rx_n - h$, $x_0 = K$ by using the following way:

   (a) Find the equilibrium x_*.

   (b) Make a change of variable $y_n = x_n - x_*$, and show that y_n satisfies $y_{n+1} = ry_n$.

   (c) Solve the equation of y_n and give the formula for x_n.

3. (a) Use Matlab to generate and plot the first 30 terms of the solutions to the difference equation
$$x_{n+1} = rx_n(1 - x_n), x_0 = 0.1$$
   with $r = 0.9, 1.5, 3, 4$.
   (b) Use Matlab to draw the cobweb maps for each case above.

4. Find the equilibria of $x_{n+1} = rx_n(1 - x_n)$ for $r = 1.5$ and $r = 4$, and determine the stability of each equilibrium.

5. Textbook page 9, 1.1.

6. For the Logistic equation $x_{n+1} = rx_n(1-x_n)$ determine at what r value the equilibrium $x = 0$ bifurcates, and at what $r > 1$ value the positive equilibrium exhibits a flip bifurcation.