

Assignment 6

Math 345, Prof. Shi

Due: Wednesday , Oct 25 (11am)

1. Use `pplane7` program to study the system:

$$x' = 4x - x^3 - xy, \quad y' = 6y - y^2 - 3xy.$$

- Show the phase portrait with equilibrium points, nullclines in an appropriate window.
 - Show the $t - x$ and $t - y$ graphs of the solution with initial condition $x(0) = 2$ and $y(0) = 3$.
 - Show the local phase portraits of each equilibrium using zoom-in function in Matlab, and identify the type of each equilibrium. (sink, source, saddle, etc.)
2. The competition between Nile.com and Narnes & Bobel can be described by a nonlinear model:

$$\begin{aligned} \frac{dx}{dt} &= 3x - 3y, \\ \frac{dy}{dt} &= -x + 2y(1 - y). \end{aligned}$$

(**We only consider** $x \geq 0, y \geq 0$.)

- Find and sketch the nullclines of the system, and mark the direction of the equation vector field on the nullclines.
 - Find all equilibrium points of the system.
 - At each equilibrium point, linearize the system and identify the type of the linearized system. (sink, source, saddle, etc.)
 - What is the outcome of competition? Describe asymptotic behavior for typical initial values.
3. A battle between two armies is modeled by the system: (where $a > 0$ is the reinforcement rate of army x)

$$x' = ax - x^3 - xy, \quad y' = 6y - y^2 - 3xy.$$

- When a is in certain range, more precisely, in an interval (a_0, a_1) , the system have two positive equilibrium points (which means (x_e, y_e) so that $x_e > 0$ and $y_e > 0$). Find this interval.
- When $a_0 < a < a_1$, sketch the phase portrait, and indicate which equilibrium is stable.