

# Assignment 7

Math 345, Prof. Shi

Due: Wednesday , Nov 8 (11am)

1. page 59 2.4(a,d) (Hint: for (a), find the coexistence steady state, and its Jacobian, then determine the type from trace-determinant; for (d), sketch the nullcline, steady state solutions, and typical orbits on the phase portrait.)
2. Schnakenberg proposes a biochemical reaction model with equation:

$$x' = x^2y - x + b, \quad y' = -x^2y + a$$

- (a) Find the steady state solution  $(x_*, y_*)$ .
  - (b) Calculate the Jacobian at  $(x_*, y_*)$ .
  - (c) Use trace-determinant of Jacobian to determine where the Hopf bifurcation occurs.
  - (d) Assume that  $a = 0.5$ , find the Hopf bifurcation value  $b_0$ .
  - (e) Use `pp1ane7` program to show the phase portraits of the system with  $a = 0.5$ ,  $b < b_0$  and  $b > b_0$ .
3. A study of Yale University freshmen described an influenza epidemic with  $S_0 = 0.911$  and  $S_\infty = 0.513$ . Here we measure the number of susceptibles as a fraction of the total population size.
    - (a) Estimate the basic reproduction number  $\beta S_0/\alpha$  and determine whether there was an epidemic.
    - (b) What fraction of students would have to be immunized to prevent an epidemic?