

Test 1/homework 5 (take home)

Math 345, Prof. Shi

Due: Friday, Oct 13 (5pm)

1. (20 pt) Consider a population governed by the following nonlinear difference equation:

$$x_{n+1} = \frac{2x_n}{1 + x_n^2}.$$

Determine the steady states and their stability. Use the cobwebbing method to sketch the approximate behavior of solutions from some initial starting value of x_0 .

2. (30 pt) A model with an Allee effect is based on the fact that unmated female must find a mate and during the time they are searching, before reproduction, they experience an increased mortality due to predation. The model for population size $N(t)$ at time t satisfies:

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K} - \frac{a}{1 + bN} \right), \quad N(0) > 0,$$

where r, a, b, K are positive parameters. We use a to refer to the Allee effect. (If $a = 0$ the model is logistic equation.

- (a) In the following table, fill in the dimensions of all parameters in terms of the dimensions of variables.

Variable	Dimension	Parameter	Dimension
t	τ	r	
N	λ	N	
		a	
		b	

- (b) Use the change of variable:

$$x = bN, \quad s = rt.$$

Derive the new equation in the new variables x and s , and the new equation is in a form:

$$\frac{dx}{ds} = x \left(1 - \frac{x}{M} - \frac{a}{1 + x} \right), \quad x(0) > 0.$$

Express M in term of old parameters.

- (c) Now we assume that $M = 2$, so the equation is

$$\frac{dx}{ds} = x \left(1 - \frac{x}{2} - \frac{a}{1 + x} \right).$$

Determine the stability of the equilibrium $x = 0$.

- (d) For $a > 0$, there are two bifurcation points where the dynamics changes. Find these two points $0 < a_1 < a_2$, describe each bifurcation, sketch the phase line before and after bifurcations, and draw a bifurcation diagram indicating the change of the equilibrium solutions.

3. (20 pt) On attached page you can find the population data of North America whooping crane. We hope to predict the population of whooping crane for the future.
- (a) Use Malthus growth model $N_{n+1} = \lambda N_n$ and Matlab program of linear regression to fit the data: determine the best fitting λ and N_0 , and find the predicted population for 2004, 2005, and 2006; graph the whooping crane data and the Malthus growth curve.
 - (b) Use logistic model or Beverton-Holt model and Matlab program to fit the data: the solution of Beverton-Holt model is $N_n = \frac{N_0 K \lambda^n}{K + N_0(\lambda^n - 1)}$, and using the change of variable $y_n = \ln(N_n^{-1} - K^{-1})$ we find $y_n = \ln(N_0^{-1} - K^{-1}) - n \ln \lambda$; use $K = 400$ and $K = 1000$ and Matlab linear regression programs to generate two estimates for N_0 and λ , and find the predicted population for 2004, 2005, and 2006; graph the whooping crane data and two Beverton-Holt growth curves.
 - (c) Conclude your research by comparing the different models used, and comment on the validity of models.
4. (10 pt) The propagation of an annual plant is governed by the second order difference equation $x_{n+2} = x_{n+1} - 6x_n$ where x_n denotes the number of plants after n -years. Find x_n given that $x_0 = 2$ and $x_1 = 4$.
5. (10 pt) Exercise 1 on page 82 of Prof. Schreiber's notes.
6. (10 pt) Exercise 4 on page 94 of Prof. Schreiber's notes.