Dimensions:

Dimension is the unit of certain type of physical quantity.

Example: the dimension of variable t (time) is second or hour, the dimension of variable L (length) is meter or foot, and the dimension of parameter N (carrying capacity) is million people.

Dimensionless: A number is dimensionless if it is just a number, it is not a measurement of any type of physical quantity.

Example: Quotient of N (carrying capacity) and P (the population variable) is a dimensionless quantity.

Every variable or parameter in the differential equation has a dimension or is dimensionless.

Nondimensionalization:

Nondimensionalization is a process of changing variables by scaling so that the new variables are dimensionless, and it leads to a simpler form of the equation with fewer parameters.

Step 1: List all variables, parameters and their dimensions.

Step 2: Take each variable and create a new variable by dividing by the combination of parameters that has the same dimension in order to create a dimensionless variable. (Usually there is more than one way to do this.

Step 3: Calculate and simplify the new equation via change of variables.

Step 4: Introduce new parameters.

Example 1:

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right)\left(\frac{P}{M} - 1\right). \tag{1}$$

Variable	Dimension	Parameter	Dimension
t	τ	k	
P	ρ	M	
		N	

Example 2:

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right) - \frac{BP^2}{A^2 + P^2}, \quad P(0) = P_0. \tag{2}$$

Variable	Dimension	Parameter	Dimension
t	τ	k	$1/\tau$
P	ρ	N	ρ
		A	ρ
		B	ρ/τ
		P_0	ρ

Harvesting, Predation, Hunting and Fishing:

Harvesting (or predation, hunting and fishing) is an effort on a population model of the removal of members of the population at a a specific rate.

The general population with harvesting is

$$\frac{dP}{dt} = f(P) - h(t, P),$$

where $f(P)$ is the growth rate of P without harvesting (which could be Malthus, logistic or allee), and $h(t, P)$ is the harvesting rate in a unit of members per unit time.

We assume that $f(P) = kP\left(1 - \frac{P}{N}\right)$.

Model 4a: Constant yield harvesting

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right) - h$$

Example: In a hunting season, only 100 deers are allowed to be hunted; by regulation of International Commission for the Conservation of Atlantic Tunas, only 2000 tons of Atlantic Tunas are allowed to be caught by commercial fishing boats.

Model 4b: Constant effort harvesting

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right) - hP$$

Example: In the modeling of fisheries, we assume that the number of fish caught per unit time is proportional to the effort expending in fishing. The fishing effort can be measured, for example, by the number of boats fishing at a give time. This is reasonable when the population of fish is large and there is no regulation to limit the fishing.

Autonomous Equation modeling harvesting:

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right) - h(P)$$

Assumptions:

$h(0) = 0$: when the population is zero, there is nothing to harvest

$h'(P) \geq 0$: when there is more can be harvested, then the harvest rate is higher

Model 4b: $h(P) = aP$, a linear function.

Model 4a: maybe not a good model, but still make sense sometime.

In this model, $P(t)$ is the population of a prey, and $h(P)$ measure the rate at which prey are taken by a predator, as a function of prey population P (this was called "functional response" of predators to prey population (Holling, 1959, 1965).)

Model 4c: Holling's type I model

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right) - h(P)$$

where $h(P) = aP$ when $0 \leq P \leq P_0$ and $h(P) = aP_0$ when $P > P_0$.

Assumptions: The number of predator is assumed to be constant, and they cannot consume more preys when P is large. Type I is unusual, except for the case of filter-feeding crustacea feeding on algar cells.

Model 4d: Holling's type II model

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right) - \frac{AP}{1+BP}$$

Assumptions: The number of predator is assumed to be constant, and they cannot consume more preys when P is large. It takes the predator a certain amount of time to kill and eat each prey. So suppose that in one hour, the predator (a wolf) can catch AP number of prey (rabbits) (it is proportional to P since when P is larger, the wolf has better chance to meet rabbits,) but it needs T hour to handle and eat each rabbit caught. So for all AP rabbits, it takes ATP hours, and in fact the wolf spends $1 + ATP$ hours on these AP rabbits. So in 1 hour, the wolf actually only eats $\dfrac{AP}{1+ATP}$ rabbits. We use $B = AT$ as a new parameter in the equation.

Model 4e: Holling's type III model

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right) - \frac{AP^n}{1 + BP^n}$$

We will introduce this model later in spruce budworm model