Summary of Part 1:

Population models for single species:

Model 1: Malthus model
Model 2: Logistic model
Model 3a,b: Allee effect
Model 4a,b,c,d,e: Harvesting model
Model 5: Spruce budworm model

Fishing model: management of renewable natural resources (fish, forest...)

<u>A General Population Model</u>: (deterministic model)

$$\frac{dP}{dt} = [B(t) - D(t)]P\left[1 - \frac{P}{N(t)}\right] + I(t) - E(t)$$

$B(t)$: birth rate per capita at t

$D(t)$: death rate per capita at t

$N(t)$: carry capacity due to technology

$I(t)$: rate of immigration

$E(t)$: rate of emigration

Stochastic model: $P(t)$ not a function, but a probability distribution

Mathematical methods for single ODE:

1. Analytic methods: separation of variables.

2. Numerical method: Euler's method, data fitting.

3. Qualitative methods: direction field, graph of solutions, phase line, equilibrium points and their stability, bifurcation.

4. Nondimensionalization.

Model 4d: Holling's type II model

$$\frac{dQ}{ds} = Q\,(1 - Q) - \frac{hQ}{1 + aQ}$$

A. when $a = 2$, $Q\,(1 - Q) - \dfrac{hQ}{1 + 2Q} = 0$

$Q = 0$ or $2Q^2 - Q + (h - 1) = 0$, $Q = \dfrac{1 \pm \sqrt{9 - 8h}}{4}$

Bifurcation diagram: $h = -2Q^2 + Q + 1$ (but in a $h - Q$ graph)

Two bifurcation points:
$h = 9/8$: subcritical saddle-node bifurcation
$h = 1$: transcritical bifurcation

Biological implications when $a = 2$:

1. When $0 < h < 1$, there are two equilibrium points, 0 and $Q_+ = 0.25(1 + \sqrt{9 - 8h})$. The system is similar to logistic equation.

2. When $1 < h < 9/8$, there are three equilibrium points, 0 and $Q_\pm = 0.25(1 + \sqrt{9 - 8h})$. The system is similar to logistic equation with allee effect.

3. When $h > 9/8$, 0 is the only equilibrium point. The population will become extinct no matter how large the initial population is.

4. Depending on the value of h, the state of the system is survival, partial survival and extinction.

B. General (a, h). $Q\,(1 - Q) - \dfrac{hQ}{1 + aQ} = 0$

$Q = 0$ or $aQ^2 + (1 - a)Q + (h - 1) = 0$,

$Q_{\pm} = \dfrac{a - 1 \pm \sqrt{(a+1)^2 - 4ah}}{2a}$, Basic border line: $h = \dfrac{(a+1)^2}{4a}$

when $0 < h < \dfrac{(a+1)^2}{4a}$, three equilibrium points

when $h = \dfrac{(a+1)^2}{4a}$, two equilibrium points (except $a = 1$)

when $h > \dfrac{(a+1)^2}{4a}$, one equilibrium points

But we also count the negative equilibrium points

Delicate but simple analysis:

when $Q_+ < 0$: $Q_+ = \dfrac{a - 1 + \sqrt{(a+1)^2 - 4ah}}{2a} < 0$

when $Q_+ \geq 0$ but $Q_- < 0$: $Q_- = \dfrac{a - 1 - \sqrt{(a+1)^2 - 4ah}}{2a} < 0$

Derivative method:

Solve $f(Q) = aQ^2 + (1-a)Q + (h-1) = 0$
and $f'(Q) = 2aQ + (1-a) = 0$.